Automatic Performance Tuning
Spiral, FFTW, ATLAS & Friends

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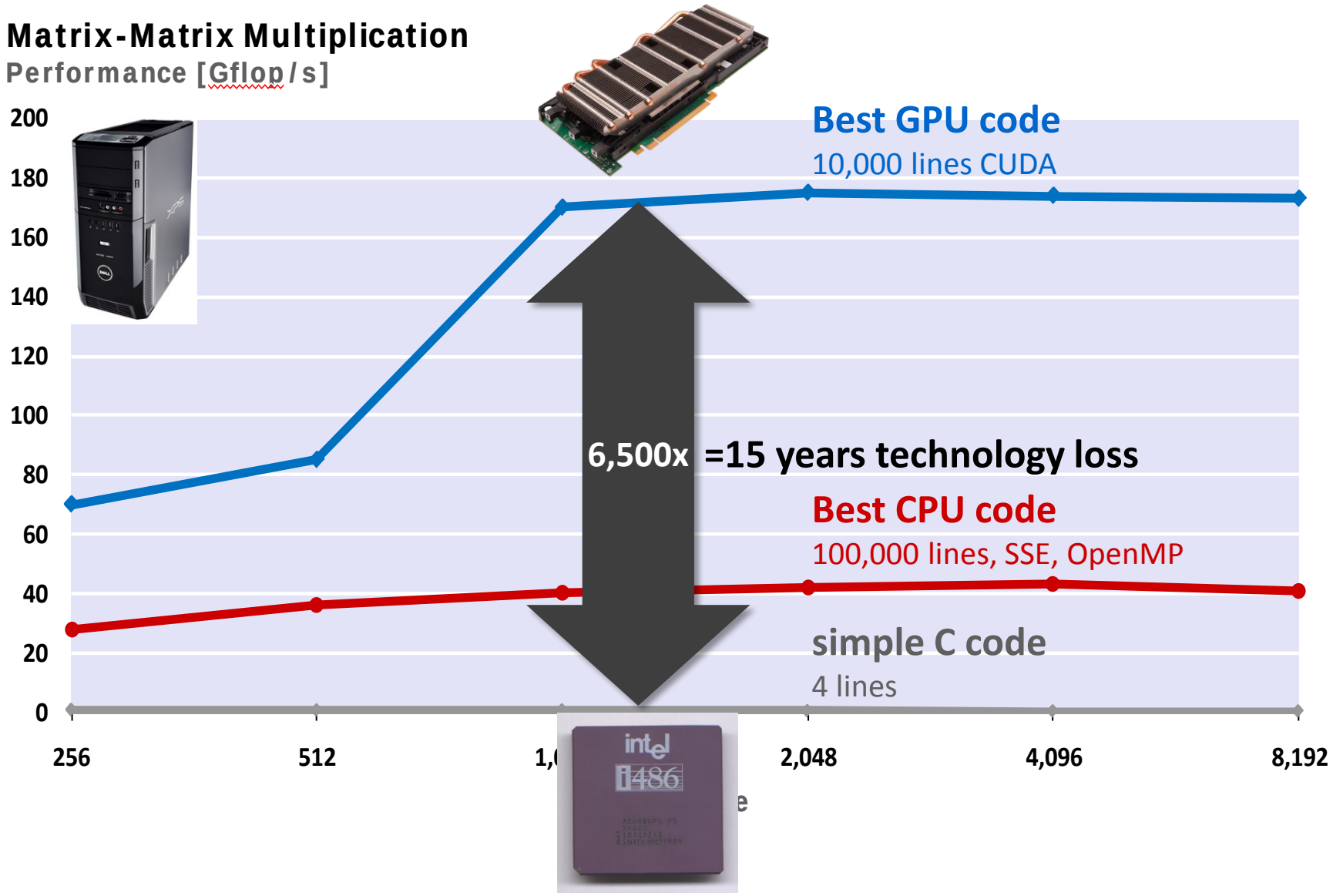
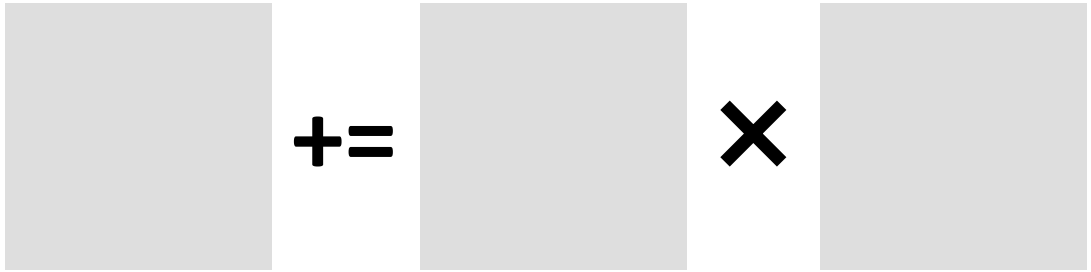
The Cost Of Portability and Maintainability



Compilers: The Fundamental Problem

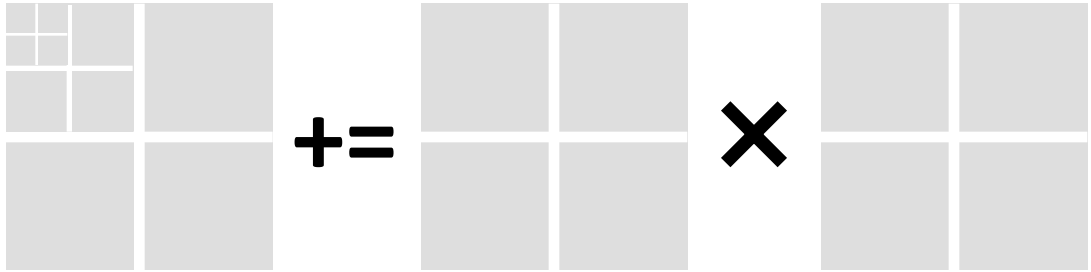
Matrix-matrix multiplication

```
for i=1:N
  for j=1:N
    for k=1:N
      C[i,j] += A[i,k]*B[k,j]
```



Tiled matrix-matrix multiplication

```
for i=1:NB:N
  for j=1:NB:N
    for k=1:NB:N
      for i0=i:NU:i+NB
        for j0=j:NU:j+NB
          for k0=k:NU:k+NB
            for k00=k0:1:k0+NU
              for j00=j0:1:j0+NU
                for i00=i0:1:i0+NU
                  C[i00,j00] +=
                    A[i00,k00]*B[k00,j00]
```

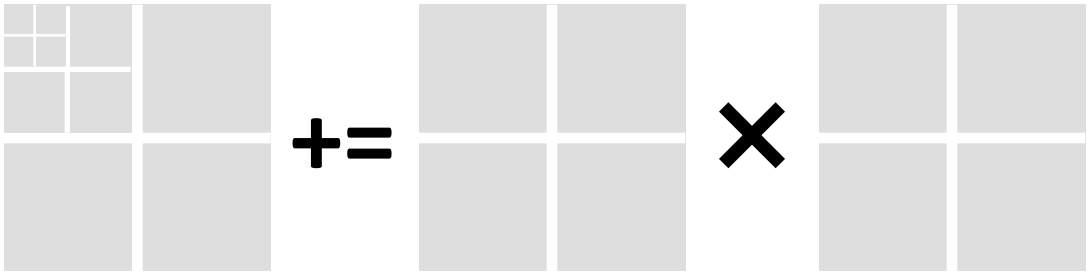


Problem: which transformation order? What parameter values?

How to Overcome Compiler Limitations?

- **Autotuning**

  Write parameterized program and empirically find good parameters



- **Program generation**

  Automatically generate big basic blocks and special case code

  Use special instructions (SSE/AVX/NEON/AltiVec/QPX,…)

```
for (i = 0; i < 4; i++)
   A[i] = B[i] * C[i];
```

```
A[0] = B[0] * C[0];
A[1] = B[1] * C[1];
A[2] = B[2] * C[2];
A[3] = B[3] * C[3];
```

When done: 30MB of source code for DGEMM

Autotuning: From Simple to "Really Hard"

- **Level 0: simple C program**
  implements the algorithm cleanly
- **Level 1: C macros plus search script**
  use C preprocessor for meta-programming
- **Level 2: scripting for code specialization**
  text-based program generation, e.g., ATLAS
- **Level 3: add compiler technology**
  internal code representation, e.g., FFTW's genfft
- **Level 4: synthesize the program from scratch**
  high level representation, e.g., TCE and Spiral

Performance Portability Landscape

Synthesis from Domain Math

- **Spiral**
  Signal and image processing, SDR
- **Tensor Contraction Engine**
  Quantum Chemistry Code Synthesizer
- **FLAME**
  Numerical linear algebra (LAPACK)

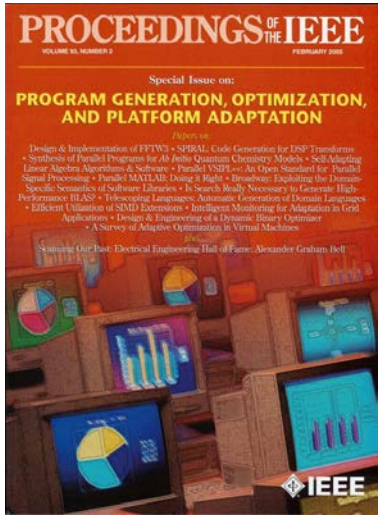
Compiler Synthesis/Autotuning

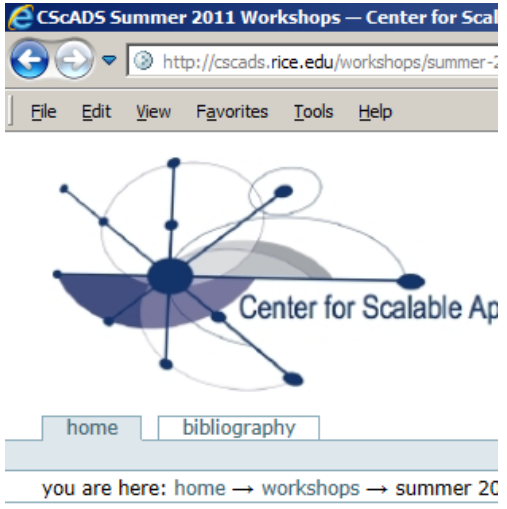
- **Polyhedral framework**
  XL C, Pluto, CHiLL
- **Transformation prescription**
  CHiLL, POET
- **Profile guided optimization**
  Intel C, IBM XL

Numerical Libraries

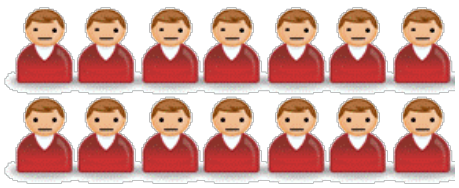
- **ATLAS**
  BLAS generator
- **FFTW**
  kernel generator
- **Vendor math libraries**
  Code generation scripts

Autotuning Primer

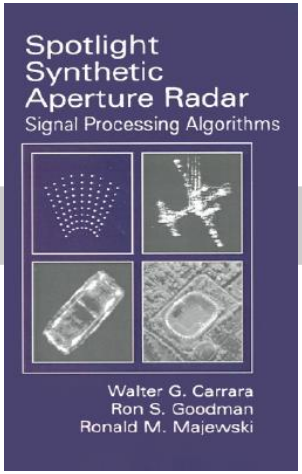




Spiral: Computer Writes Best SAR Code



Programming



Synthesis

Result

Same performance, 1/10th human effort, non-expert user

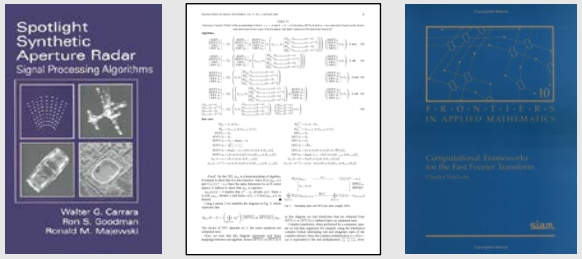
Key ideas

restrict domain, use mathematics, performance portability

D. McFarlin, F. Franchetti, M. Püschel, and J. M. F. Moura: **High Performance Synthetic Aperture Radar Image Formation On Commodity Multicore Architectures.** in Proceedings SPIE, 2009.

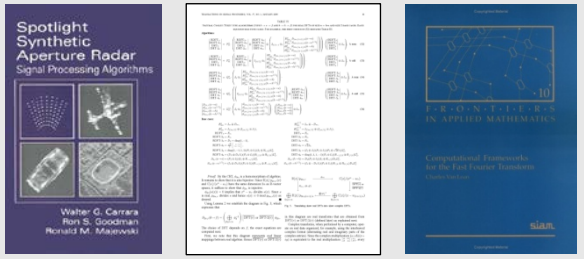
What is Spiral?

Traditionally



High performance library optimized for given platform

Spiral Approach



Spiral

High performance library optimized for given platform

Comparable performance

Algorithms: Rules in Domain Specific Language

Linear Transforms

$$
\begin{aligned}
\mathbf{DFT}_n &\rightarrow (\mathbf{DFT}_k \otimes \mathrm{I}_m)\, \mathrm{T}^n_m (\mathrm{I}_k \otimes \mathbf{DFT}_m)\, \mathrm{L}^n_k, \quad n = km \\
\mathbf{DFT}_n &\rightarrow P_n (\mathbf{DFT}_k \otimes \mathbf{DFT}_m) Q_n, \quad n = km,\ \gcd(k,m) = 1 \\
\mathbf{DFT}_p &\rightarrow R_p^T (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) D_p (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) R_p, \quad p \text{ prime} \\
\mathbf{DCT\text{-}3}_n &\rightarrow (\mathrm{I}_m \oplus \mathrm{J}_m)\, \mathrm{L}^n_m (\mathbf{DCT\text{-}3}_m(1/4) \oplus \mathbf{DCT\text{-}3}_m(3/4)) \\
&\quad \cdot (\mathrm{F}_2 \otimes \mathrm{I}_m) \begin{bmatrix} \mathrm{I}_m & 0 \oplus -\mathrm{J}_{m-1} \\ & \frac{1}{\sqrt{2}}(\mathrm{I}_1 \oplus 2\,\mathrm{I}_m) \end{bmatrix}, \quad n = 2m \\
\mathbf{DCT\text{-}4}_n &\rightarrow S_n \mathbf{DCT\text{-}2}_n\, \mathrm{diag}_{0 \le k < n}(1/(2\cos((2k+1)\pi/4n))) \\
\mathbf{IMDCT}_{2m} &\rightarrow (\mathrm{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathrm{J}_m) \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m \right) \oplus \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m \right) \mathrm{J}_{2m}\, \mathbf{DCT\text{-}4}_{2m} \\
\mathbf{WHT}_{2^k} &\rightarrow \prod_{i=1}^{t} (\mathrm{I}_{2^{k_1+\cdots+k_{i-1}}} \otimes \mathbf{WHT}_{2^{k_i}} \otimes \mathrm{I}_{2^{k_{i+1}+\cdots+k_t}}), \quad k = k_1 + \cdots + k_t \\
\mathbf{DFT}_2 &\rightarrow \mathrm{F}_2 \\
\mathbf{DCT\text{-}2}_2 &\rightarrow \mathrm{diag}(1, 1/\sqrt{2})\, \mathrm{F}_2 \\
\mathbf{DCT\text{-}4}_2 &\rightarrow \mathrm{J}_2\, \mathrm{R}_{13\pi/8}
\end{aligned}
$$

Viterbi Decoding

$$
\begin{aligned}
\underbrace{\mathbf{Vit}}_{\mathrm{vec}(v)} &\rightarrow \underbrace{\left(\prod (L \times I) \circ (I \otimes C)\right) \circ Id}_{\mathrm{vec}(v)} \\
&\rightarrow \left(\prod \underbrace{(L \times I) \circ (I \otimes C)}_{\mathrm{vec}(v)} \right) \circ Id \\
\times \quad &\rightarrow \left(\prod (L \otimes I_v \times I) \circ (I \otimes C \otimes I_v) \circ (\vec{L} \times I)\right) \circ Id \\
&\rightarrow \prod (L \otimes I_v \times I) \circ (I \otimes (B \otimes I_v)) \circ (\vec{L} \times I)
\end{aligned}
$$

Matrix-Matrix Multiplication

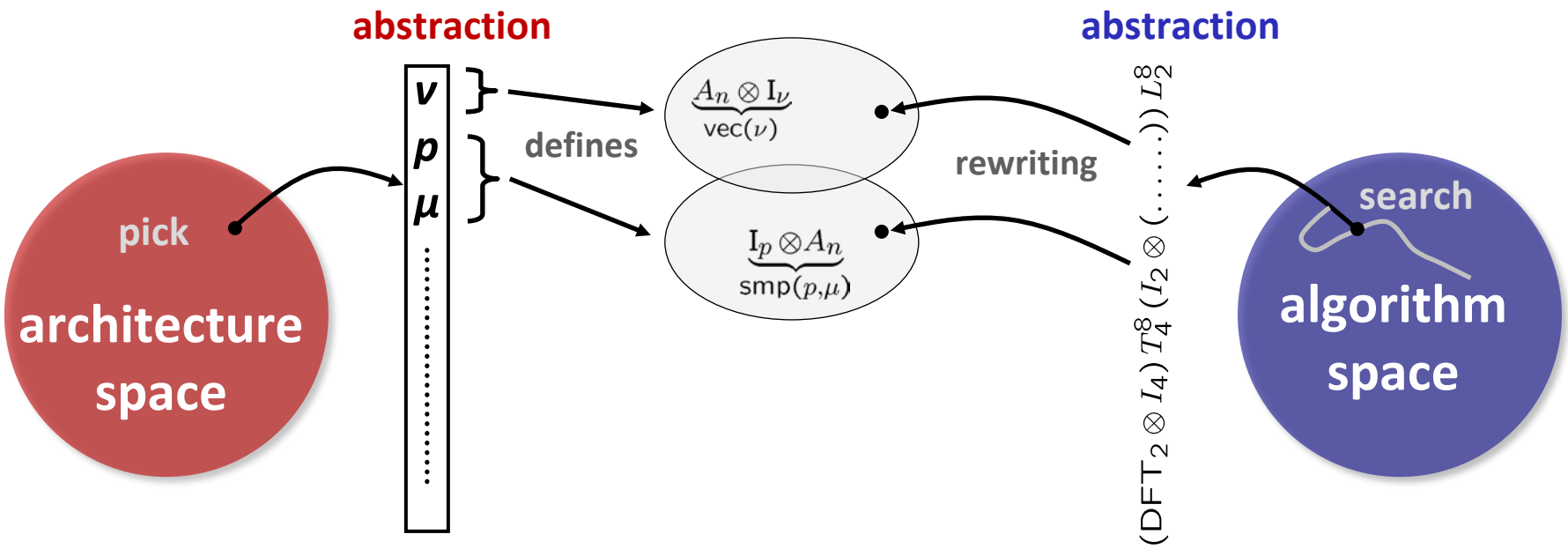
$$
\begin{aligned}
&\mathrm{MMM}_{1,1,1} \rightarrow (\cdot)_1 \\
&\mathrm{MMM}_{m,n,k} \rightarrow (\otimes)_{m/m_b \times 1} \otimes \mathrm{MMM}_{m_b,n,k} \\
&\mathrm{MMM}_{m,n,k} \rightarrow \mathrm{MMM}_{m,nb,k} \otimes (\otimes)_{1 \times n/nb} \\
&\mathrm{MMM}_{m,n,k} \rightarrow ((\Sigma_{k/k_b} \circ (\cdot)_{k/k_b}) \otimes \mathrm{MMM}_{m,n,k_b}) \circ \\
&\qquad\qquad ((L^{mk/k_b}_{k/k_b} \otimes I_{k_b}) \times I_{kn}) \\
&\mathrm{MMM}_{m,n,k} \rightarrow (L^{mn/n_b}_m \otimes I_{n_b}) \circ \\
&\qquad ((\otimes)_{1 \times n/n_b} \otimes \mathrm{MMM}_{m,n_b,k}) \circ \\
&\qquad\qquad (I_{km} \times (L^{kn/n_b}_{n/n_b} \otimes I_{n_b}))
\end{aligned}
$$

Synthetic Aperture Radar (SAR)

$$
\begin{aligned}
\mathrm{SAR}_{k \times m \rightarrow n \times n} &\rightarrow \mathbf{DFT}_{n \times n} \circ \mathrm{Interp}_{k \times m \rightarrow n \times n} \\
\mathbf{DFT}_{n \times n} &\rightarrow (\mathbf{DFT}_n \otimes \mathrm{I}_n) \circ (\mathrm{I}_n \otimes \mathbf{DFT}_n) \\
\mathrm{Interp}_{k \times m \rightarrow n \times n} &\rightarrow (\mathrm{Interp}_{k \rightarrow n} \otimes_i \mathrm{I}_n) \circ (\mathrm{I}_k \otimes_i \mathrm{Interp}_{m \rightarrow n}) \\
\mathrm{Interp}_{r \rightarrow s} &\rightarrow \left(\bigoplus_{i=0}^{n-2} \mathrm{InterpSeg}_k \right) \oplus \mathrm{InterpSegPruned}_{k,\ell} \\
\mathrm{InterpSeg}_k &\rightarrow \mathrm{G}^{u \cdot n \rightarrow k}_f \circ \mathrm{iPrunedDFT}_{n \rightarrow u \cdot n} \circ \left(\frac{1}{n}\right) \circ \mathbf{DFT}_n
\end{aligned}
$$

Spiral's Domain-Specific Program Synthesis

Model: common abstraction
= spaces of matching formulas



Architectural parameter:
Vector length,
#processors, …

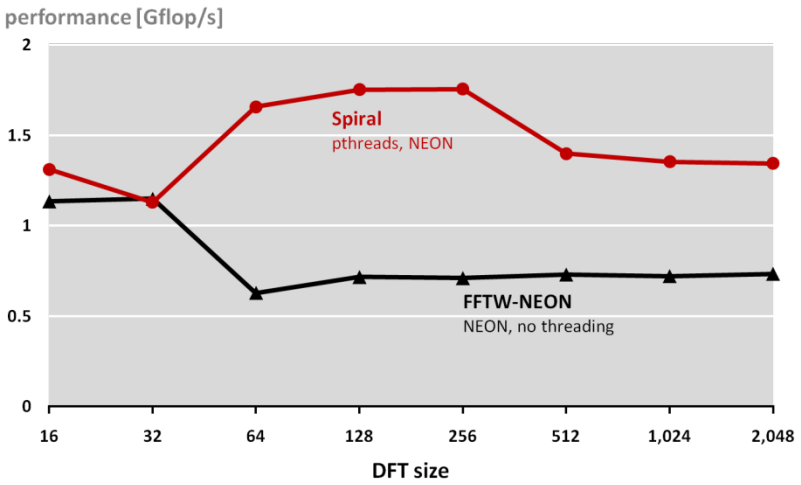
optimization

Kernel:
problem size,
algorithm choice

From Cell Phone To Supercomputer

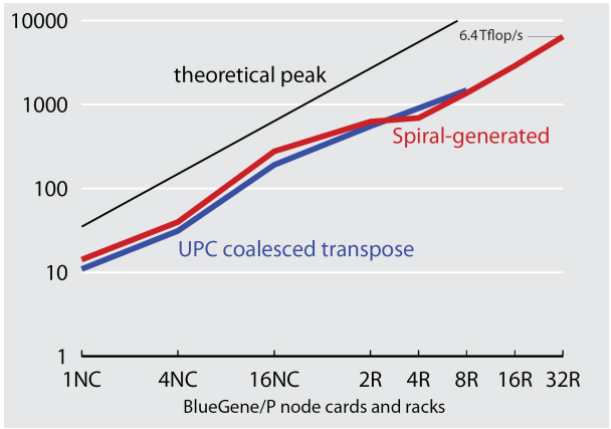
DFT on Samsung Galaxy S II

Dual-core 1.2 GHz Cortex-A9 with NEON ISA



Global FFT (1D FFT, HPC Challenge)

performance [Gflop/s]



6.4 Tflop/s on BlueGene/P

Samsung i9100 Galaxy S II

Dual-core ARM at 1.2GHz with NEON ISA

SIMD vectorization + multi-threading

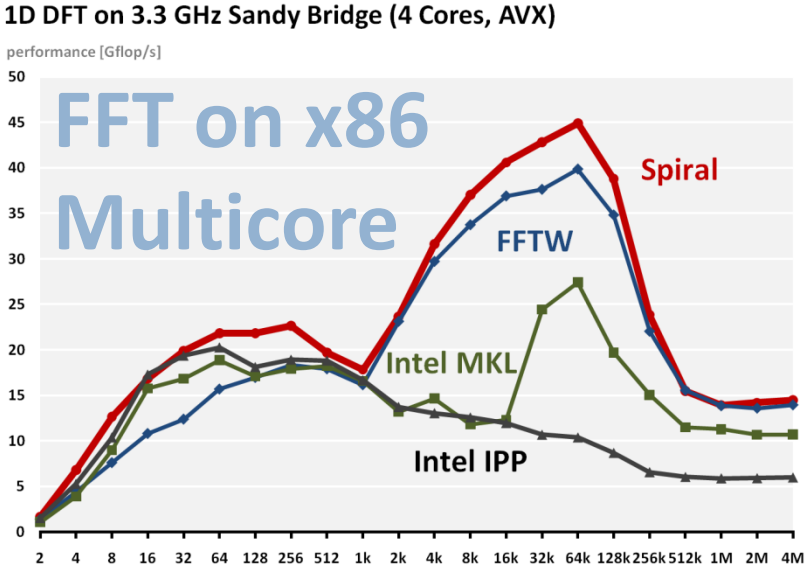
BlueGene/P at Argonne National Laboratory

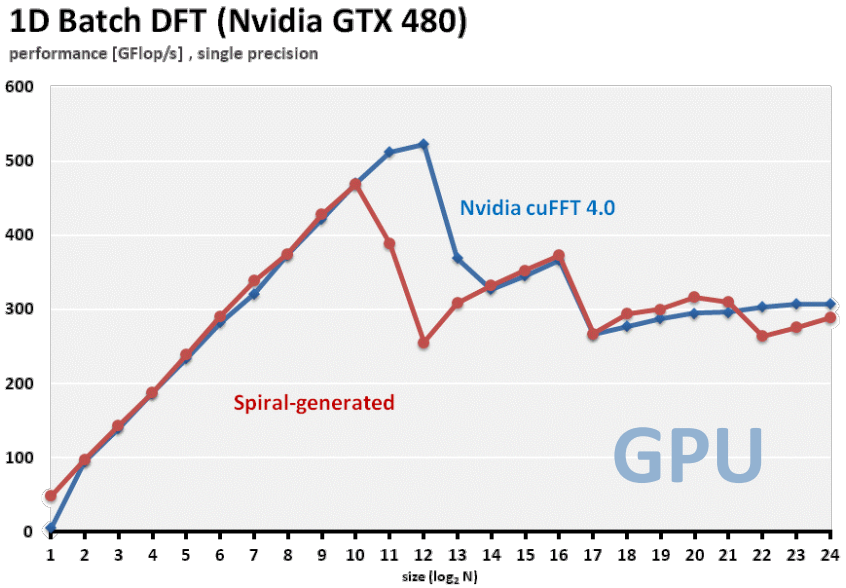
128k cores (quad-core CPUs) at 850 MHz

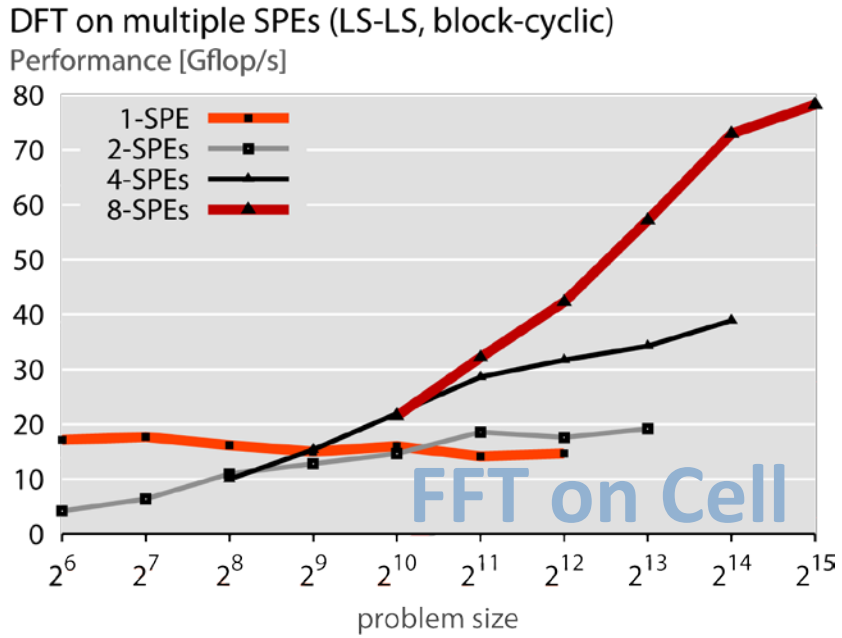
SIMD vectorization + multi-threading + MPI

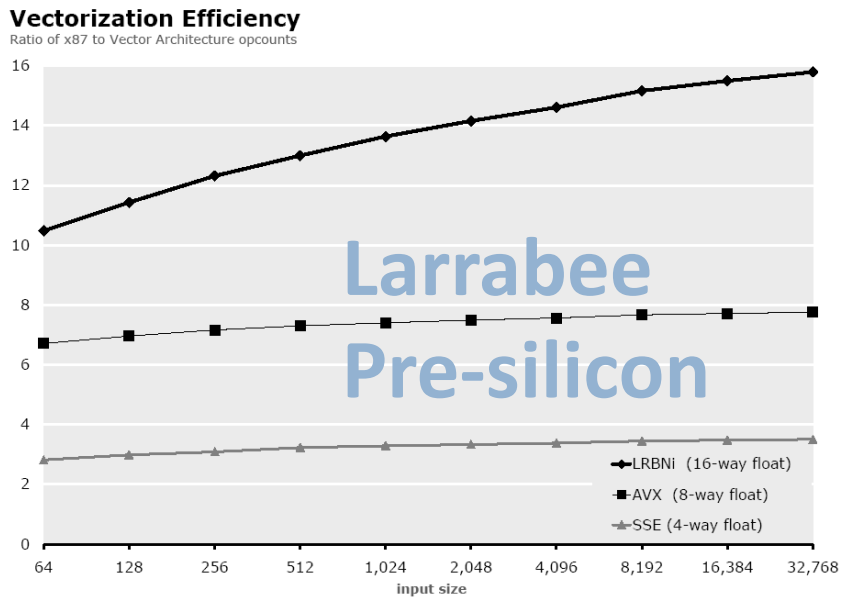
G. Almási, B. Dalton, L. L. Hu, F. Franchetti, Y. Liu, A. Sidelnik, T. Spelce, I. G. Tānase, E. Tiotto, Y. Voronenko, X. Xue:
2010 IBM HPC Challenge Class II Submission. Winner of the 2010 HPC Challenge Class II Award (Most Productive System).

Results: Spiral Outperforms Humans





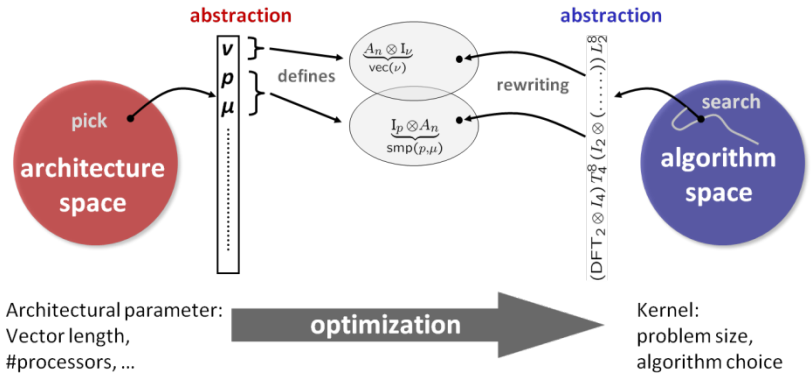




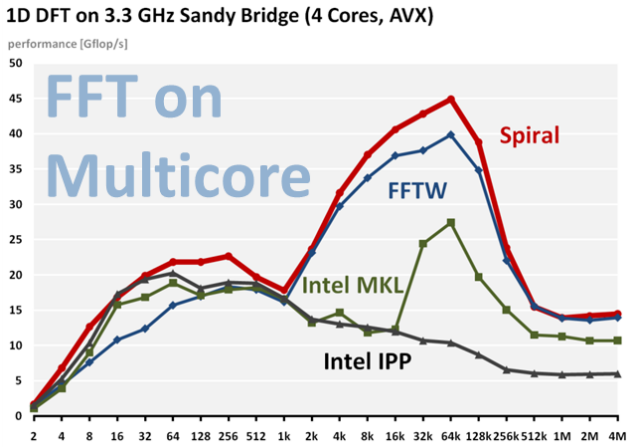
Summary: Spiral in a Nutshell

Joint Abstraction

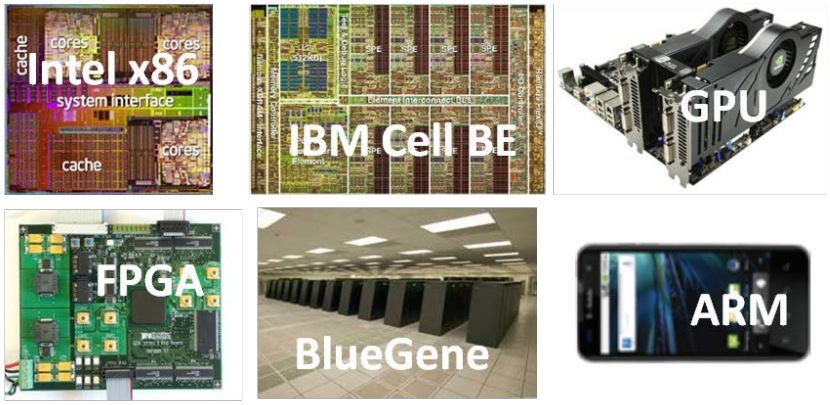
Model: common abstraction
= spaces of matching formulas



Outperforms Human Experts



Performance Portability



Application Domains

Signal Processing

$$\mathrm{DFT}_n \;\rightarrow\; (\mathrm{DFT}_k \otimes \mathrm{I}_m)\, \mathrm{T}^n_m (\mathrm{I}_k \otimes \mathrm{DFT}_m)\, \mathrm{L}^n_k,$$

Matrix Algorithms

Software Defined Radio

Image Formation (SAR)

Academic @ CMU: www.spiral.net
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